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# Warranty cost analysis: quasi-renewal inter-repair times

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## Abstract

**Purpose** – To provide a brief introduction to warranty analysis and a classification of general repairs. To introduce the notion of accelerated probability distribution and use it to model imperfect warranty repairs.

**Design/methodology/approach** – The notion of accelerated probability distribution is discussed and its similarity with quasi-renewal and geometric processes is observed. An approach to modeling imperfect warranty repairs based on the accelerated probability distributions is presented, and the corresponding expected warranty cost over the warranty period under non-renewing free replacement warranty policy is evaluated.

**Findings** – It is observed that quasi-renewal and the geometric processes are equivalent. Using data from an existing warranty database it is shown that the inter-repair times form a quasi-renewal process. The corresponding expected warranty cost over the warranty period under a non-renewing free replacement warranty policy is evaluated.

**Research limitations/implications** – This approach is applicable only if the cost of the warranty repair is an increasing function of the number of repairs.

**Practical implications** – Provides a useful approach to modeling inter-repair times incorporating the idea of imperfect repairs in practice.

**Originality/value** – Provides an approach to model imperfect warranty repairs and to evaluate the corresponding expected warranty cost.

**Keywords** Warranties, Distribution functions, Replacement costs

**Paper type** Research paper

## 1. Introduction

A product warranty is an agreement offered by a producer to a consumer to replace or repair a faulty item, or to partially or fully reimburse the consumer in the event of a failure.

The form of reimbursement of the customer on failure of an item or dissatisfaction with service is one of the most important characteristics of warranty. There are two types of warranty coverage used in industry and discussed in the literature:

(1) *Non-renewing warranty*. A newly sold item is covered by a warranty for some calendar time of duration $T$, according to certain warranty agreements. The warranter assumes all, or a portion, of the expenses associated with the failure of the product from the start of its usage, say $t_0$ (usually $t_0 = 0$), until the expiration of the warranty coverage.

(2) *Renewing warranty*. The warranter agrees to repair or replace any failed item up to time $T$ (the length of the warranty period), from the time of purchase, at no

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cost to the consumer. At the time of each repair within an existing warranty the item is warranted anew for a period of length $T$.

Warranties are very widespread and serve many purposes, including protection for the producer, the seller, and the consumer: they are used as signals of quality and as elements of marketing strategies. A general treatment of warranty analysis is given by Blischke and Murthy (1993) and Chukova *et al.* (1993), and a recent overview is given by Blischke and Murthy (1996).

From the buyer's point of view, the main role of a warranty in these transactions is protectional. Specifically, the warranty assures the buyer that a faulty item will be either repaired or replaced at no cost or at a reduced cost. A second role of warranty is informational, since it may specify appropriate use, and may be looked as an indication of quality.

One of the main roles of warranty from the producer's point of view is also protectional. Warranty terms may and often do specify the use and conditions of use for which the product is intended and provide for limited coverage or no coverage at all in the event of misuse of the product.

A second important purpose of warranty for the seller is promotional. As buyers often infer a more reliable product when a long warranty is offered, this has been used as an effective advertising tool. In addition, warranty has become an instrument, similar to product performance and price, used in competition with other manufacturers in the marketplace.

Despite the fact that warranties are so commonly used, the accurate pricing of warranties in many situations remains an unsolved problem. This may seem surprising since the fulfillment of warranty claims may cost companies large amounts of money. Underestimating true warranty costs will result in losses for a company: on the other hand, overestimating them will result in uncompetitive product prices.

The data relevant to the modeling of warranty costs in a particular industry are usually highly confidential, since they are commercially sensitive. Much warranty analysis therefore takes place in internal research divisions in large companies.

The common warranty parameters of interest to be analyzed and evaluated are the expected total warranty cost and the warranty cost per unit time over the warranty period for a particular item, as well as the lifecycle of the product. These quantities summarize the financial risk or burden carried by buyers, sellers and decision makers.

The evaluation of the parameters (e.g. warranty period or price) of the warranty program can be obtained, using appropriate models, from the producer's, seller's, buyer's and decision-maker's point of view. The values of the parameters result from the application of analytical or approximation methods, often calculated in combination with an optimization problem. Due to the complexity of the models, it is almost always necessary to resort to numerical methods, since analytical solutions exist only in the simplest situations.

The outline of this paper is as follows: In Section 2 we present a classification of types of repair. We define the accelerated lifetime distribution function in Section 3, show how they can be used to model inter-repair time in warranty settings and provide a real data example in Section 4. Section 5 concludes this study with final remarks.

## 2. Degree of repair

The evaluation of the warranty cost or any other parameter of interest in modeling warranties depends on the failure and repair processes and on the assigned preventative warranty maintenance for the items. The repairs can be classified according to the degree to which they restore the ability of the item to function (Pham and Wang, 1996). Post-failure repairs affect repairable products in one of the following ways:

- *Improved repair.* A repair brings the product to a state *better* than when it was initially purchased. This is equivalent to the replacement of the faulty item by a new and improved item.
- *Complete (or perfect) repair.* A repair completely resets the performance of the product so that upon restart the product operates as a new one. This type of repair is equivalent to a replacement of the faulty item by a new one, identical to the original.
- *Imperfect repair.* A repair contributes to some noticeable improvement of the product. It effectively sets back the clock for the repaired item. After the repair the performance and expected lifetime of the item are as they were at an earlier age.
- *Minimal repair.* A repair has no impact on the performance of the item. The repair brings the product from a "down" to an "up" state without affecting its performance.
- *Worse repair.* A repair contributes to some noticeable worsening of the product. It effectively sets forward the clock for the repaired item. After the repair the performance of the item is as it would have been at a later age.
- *Worst repair.* A repair accidentally leads to the product's destruction.

What could be the reason for imperfect, worse or worst repair? Some possible reasons are (see also Murthy and Djamaludin, 2000; Pham and Wang, 1996):

- incorrect assessment of the faulty item;
- while repairing the faulty part, damage is caused to the adjacent parts or subsystems of the item;
- partial or incorrect repair of the faulty part; and
- replacement with faulty or incompatible parts.

The type of the repair which takes place depends on the warranty reserves, related costs, assigned warranty maintenance, reliability and safety requirements of the product. The existence of extended warranty or any additional agreements in the warranty contract may influence the degree of the repair to be performed on the faulty item under warranty. Mathematically the degree of repair can be modeled through different characteristics of the lifetime distribution of the item, such as the mean total time to failure, the failure rate function or the cumulative distribution function, as in Chukova *et al.* (2004). More sophisticated techniques involving stochastic processes can be used to model the virtual age of the product and its dependence on the degree of repair. A summary of methods for modelling imperfect maintenance is given by Pham and Wang (1996).

In what follows we discuss an approach to model the effect of imperfect repair on the lifetime of the repaired item. The repairs are assumed to be instantaneous. We model the lifetime $X$ of an item in service as a random variable with cumulative distribution function (CDF) $F(x)$ and probability density function (PDF) $f(x)$. We often work with the failure rate function $\lambda(x)$, which is defined by:

$$\lambda(x) = \frac{f(x)}{\bar{F}(x)} = -\frac{\bar{F}'(x)}{\bar{F}(x)} = -\frac{d}{dx}\ln \bar{F}(x),$$

where $\bar{F}(x) = 1 - F(x)$ is the survival function. It follows that:

$$\bar{F}(x) = \exp\left(\int_0^x \lambda(u)\, du\right).$$

The mean time to failure (MTTF), $\mu = E[X]$, is often of primary interest to us.

The following notations will be used:

- $X_1$ is a random variable (RV) representing the initial lifetime of the item with CDF $F_1(x)$, PDF $f_1(x)$, failure rate function $\lambda_1(x) = [f_1(x)]/[F_1(x)]$ and $E[X_1] = \mu_1$, equivalent to the first inter-repair time; and
- $X_k$ is a RV representing the lifetime of the item after the $(k-1)$th repair with CDF $F_k(x)$, PDF $f_k(x)$, failure rate function $\lambda_k(x) = [f_k(x)]/F_k(x)$ and $E[X_k] = \mu_k$, $k = 2, 3, \ldots$; $X_k$ is also the $k$th interrepair time.

If $\{X_k\}_{k=1}^{\infty}$ are IID random variables, the inter-repair process is a renewal process and the rectifications are done by a complete repair. We are interested in methods for modeling the repair process preserving the independence between the $X_i$s and weakening the assumption of them being identically distributed. In other words, we will assume that the $F_k(x)$s are not identical, i.e. the repair results in an item with a lifetime distribution that is different from the original.

## 3. Accelerated lifetime distribution functions

In Chukova *et al.* (2004), the delayed distribution functions are introduced and studied. These functions are used to model the consecutive inter-repair times. Using two different diagnostics, namely the mean time to failure and the failure rate functions of the inter-repair times, a classification of the imperfect repairs is presented.

In what follows, we will review the notion of accelerated lifetime distribution functions (Chukova *et al.* 2004), and discuss their application to modelling imperfect repairs. Moreover, we will observe that using the accelerated lifetime distribution functions with parameter $s > 0$ to model the inter-repair times is equivalent, firstly, to assuming that the inter-repair times form a quasi-renewal process with parameters $X_1$ and ratio $r$ and, secondly, equivalent to assuming that the inter-repair times form a geometric process with parameters $X_1$ and ratio $s$, $r = 1/s$.

The quasi-renewal process is defined as follows (Wang and Pham, 1996).

*3.1 Definition*

Let $\{N(t),\ t > 0\}$$ be a counting process and $X_i$ be the $i$th inter-event time, for $i = 1, 2, \ldots$. The counting process $N(t)$ is a quasi-renewal process with parameters $X_1$ and $r > 0$, if $X_i = r^{i-1}Z_i$, $i = 1, 2, \ldots$ and $\{Z_i\}_1^{\infty}$ forms a renewal process.

Under accelerated life models, often used in survival analysis, the repaired unit usually has a lifetime distribution in the same family as the original, but a multiplicative scale factor is inserted to rescale the time argument $x$.

How should the value of $s$ be chosen? It could appear as a multiplicative factor after each repair and $\bar{F}_k(x) = \bar{F}_1(s^{k-1}x)$, i.e. each repair affects the item in an identical way and:

$$\bar{F}_{k=}(x)\bar{F}_1(s^{k-1}x), \quad k = 1, 2, \ldots \tag{1}$$

Thus $\bar{F}_2(x) = \bar{F}_1(sx)$ is the accelerated distribution function of the second inter-repair time with MTTF $\mu_2 = (1/s)\mu_1$ and a failure rate function is $\lambda_2(x) = s\lambda_1(sx)$. Using equation (1), it follows that:

$$\mu_k = \frac{1}{s^{k-1}} \quad \text{and} \quad \lambda_k(x) = s^{k-1}\lambda_1(s^{k-1}x). \quad k = 1, 2, \ldots$$

If the parameter $s > 1$, then each time the item is repaired it becomes more prone to failure and it models imperfect repairs. Perhaps the act of repair introduces a degradation. The mean time to failure becomes shorter, $\mu_{k+1} < \mu_k$. The probability that the item survives to a fixed age decreases with the number of repairs, i.e. at any $x$, $\bar{F}_1(x) > \bar{F}_2(x) > \bar{F}_3(x)\ldots$, which are shown in Figure 1.

If the parameter $0 < s < 1$, the item becomes less prone to failure each time it is repaired, i.e. the item is improved at each repair and it models improved repairs. The mean time to failure becomes longer, i.e. $\mu_{k+1} > \mu_k$. The probability that the item survives to a fixed age increases with the number of repairs, i.e. $\bar{F}_1(x) < \bar{F}_2(x) < \bar{F}_3(x)\ldots$, as shown in Figure 2. If the parameter $s = 1$, the corresponding inter-repair process is a renewal process and the repairs are complete repairs.

It could be that the first replacement causes a degradation, and all subsequent replacements share the same accelerated distribution. Then the $k$th item lifetime has a survival function $\bar{F}_k(x) = \bar{F}_1(sx)$ for $k > 1$. This model is not studied further in this paper.

The idea of the accelerated lifetime distribution functions with parameter $s$ coincides with the distribution functions of the inter-repair time in a geometric process

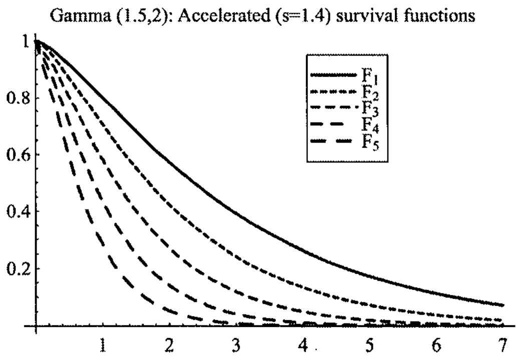


**Figure 1.**
Consecutive survival functions for $s > 0$

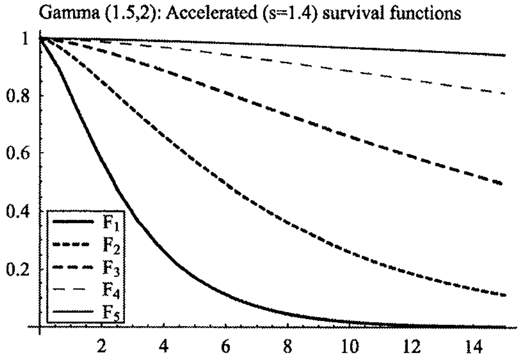


**Figure 2.**
Consecutive survival functions for $0 < s < 1$

with a ratio $s$ (see Lam, 1988), or a quasi-renewal process with a ratio $r = 1/s$ (see Wang and Pham, 1996).

If $s = 1$, the corresponding processes become ordinary renewal processes.

## 4. Warranty cost analysis

In this section we present a particular warranty model and derive the expected warranty cost for non-renewing free replacement warranty policy over warranty period of length $T$.

We make the following assumptions:

(1) The inter-repair times $\{X_i\}_1^\infty$ form a quasi-renewal process with parameters $X_1$ and a ratio $r$. The corresponding counting process is $\{N(t),\ t > 0\}$ with a quasi-renewal function $H(T) = E[N(T)]$.

(2) The repairs are instantaneous.

(3) The cost of the $i$th repair is $c_i = a + (i-1)Y$, where $Y$ is a random variable with $E[Y] = b$ (see Pham and Wang, 2001), i.e., the repair cost is an increasing function of the number of repairs.

*4.1 Expected cost over* $[0,T)$

According to our assumptions, $\{N(T),\ T > 0$ represents the total number of quasi-renewals that has occurred in the interval $[0,T)$ with $H(T) = E[N(T)]$. The distribution of $X_1$, the first inter-arrival time and the ratio $r$ of the quasi-renewal process uniquely determine its quasi-renewal function (see Wang and Pham, 1996). We have:

$$H(T) = E[N(T)] = \sum_{n=1}^{\infty} G_n(T), \qquad (2)$$

where $G_n(t)$ is the convolution of $F_1(t), F_2(t), \ldots, F_n(t)$. It has been shown (Rehmert, 2000) that there is an interesting relationship between the Laplace transforms of the quasi-renewal function and the distribution of $X_1$.

Denote by $C(t)$ the total warranty cost accumulated up to time $t$. Then, the expected warranty cost over $[0,T)$, as in Pham and Wang (2001), is equal to:

$$E[C(T)] = E\left\{\sum_{i=1}^{N(T)}[a=(i-1)Y]\right\}$$

$$=\sum_{i=1}^{\infty}E\left\{\sum_{i=1}^{n}[a+(i-1)Y]|N(T)=N\right\}P[N(T)=n]$$

$$=\sum_{i=1}^{\infty}\left\{\sum_{i=1}^{\infty}[a+(i-1)b]\right\}P[N(T)=n]$$

$$=\frac{1}{2}\sum_{i=1}^{\infty}[n(2a-b+bn)]P[N(T)=n]$$

$$=\frac{1}{2}(2a-b)E[N(T)]+\frac{b}{2}E[N^2(T)]$$

$$=\frac{1}{2}(2a-b)H(T)+\frac{b}{2}\{\mathrm{var}[N(T)]+[H(T)]^2\}. \qquad (3)$$

Thus, in order to evaluate the expected warranty cost over the warranty period, we need to compute the value of the quasi-renewal function $H(T)$ and the variance of $N(T)$. It can be shown (Wang and Pham, 1996) that the variance of $N(T)$ is:

$$\mathrm{var}[N(T)] = \sum_{n=1}^{\infty}(2n-1)G_n(T). \qquad (4)$$

Even though equations (2) and (4) provide an analytical form for the quantities needed to evaluate the expected warranty cost over the warranty period, the actual calculations usually require the usage of numerical methods.

*4.2 Example: expected cost over* $[0,T)$

We will study the inter-repair time of a particular 2001 model vehicle.

We have extracted out of a large warranty database a particular vehicle, identified by its unique identification number (VID), such that it has six claims within the warranty period. Table I summarizes our data set. The fourth column of Table I classifies the repairs by the subsystem of the vehicle they have affected and is given by the "labor option" of the repair. For confidentiality reasons the interpretation of "labor options" is not given. In our further study we do not make a difference between the repairs based on the labor options.

Table I presents only a part of the warranty record of this vehicle in the database. A number of additional parameters, related to the age of the vehicle at the time of the claim and others, are available, but they are not of interest in our current study. The costs in column 3 are slightly modified in order to fit assumption 3 regarding the cost of the $i$th warranty repair.

In the auto industry, the "time" parameter allows for two different interpretations. Firstly, it can be thought as the age of the vehicle, i.e. the time in service. Secondly, it can be measured by the mileage accumulation, which is the same as the usage of the vehicle. In this study we take the "time" parameter as the usage of the vehicle, i.e. the inter-repair "time" is the mileage accumulation between consecutive repairs, and the length of the warranty period is $T = 36,000$ (miles).

Next, we check (Leung and Lee, 1998) whether a quasi-renewal process is an appropriate model for the data $\{X_i\}_1^6$given in the second column of Table I, and compute the quantities needed to evaluate the expected warranty cost over the warranty period.

*4.2.1 Testing whether the quasi-renewal process is a good fit for the data in Table I*

*4.2.1.1 Testing for trend in the data set.* We will use the Laplace test (see Ascher and Feingold, 1984), to test the following hypotheses:

$H_O$. $X_i$s are identically distributed.

$H_a$. $X_i$s are not identically distributed.

Assuming a significance level $\alpha = 0.05$, the computed value of the test statistics U= $-2.64472$ and the rejection region ($U > 1.96$ or $-1.96 < U$) lead to the rejection of $H_O$, i.e. there is a trend in the data.

*4.2.1.2 Testing whether the data come from a quasi-renewal process.* Recall that $\{Z_i\}_1^\infty$is the renewal process related to the quasi-renewal process $\{X_i\}_1^\infty$ (see the definition). Let $\mu_{\ln(Z)} = E[\ln(Z_i)]$ for $i = 1, 2, ,\ldots$. $Z_i$s are the inter-event times of a renewal process, i.e. they are IID random variables. Thus, they can be presented in the form:

$$\ln(Z_i) = \mu_{\ln(Z)} + e_i, \quad i = 1, 2, \ldots, 6, \tag{5}$$

where $e_i$s are also IID with mean 0 and variance $\sigma_e^2$. Let $s = 1/r$ where $r$ is the ratio of the quasi-renewal model we are fitting to the data, i.e.:

$$Z_i = s^{i-1}X_i, \quad 1, 2, \ldots, 6. \tag{6}$$

One possible approach to checking whether the data is coming from a quasi-renewal process is to use a linear regression technique. As in Lam (1992), using equation (6) we get:

$$\ln(Z_i) = (i - 1)\ln(s) + \ln(Xi), \quad 1, 2, \ldots, 6. \tag{7}$$

| VID | Mileage | Cost | LaborOps | State | Country |
|---|---|---|---|---|---|
| AB1268 | 1,048 | 9.63 | A0005 | AL | USA |
| AB1268 | 3,450 | 17.78 | P1004 | AL | USA |
| AB1268 | 4,603 | 25.93 | Y1044 | AL | USA |
| AB1268 | 7,742 | 34.08 | V0202 | AL | USA |
| AB1268 | 14,426 | 42.23 | Z4512 | AL | USA |
| AB1268 | 32,866 | 50.38 | P2316 | AL | USA |

**Table I.**
The structure of the warranty database

Thus, from equations (5) and (7) we obtain:

$$\ln(X_i) = [\mu_{\ln(Z)} + \ln(s)] - i\ln(s) + e_i, \quad i = 1, 2, \ldots, 6, \tag{8}$$

which is obviously a linear regression equation. Therefore, because the plot of $\ln(X_i)$ against $i$ suggests a linear relationship between them (see Figure 3), we can accept that the dataset comes from a quasi-renewal process. Using equation (8), we find the least-squares estimate $\hat{s} = 0.532749$, corresponding to $\hat{r} = 1.87706$. Moreover, we will distinguish a quasi-renewal process from a renewal process by testing at $\alpha = 0.01$ the following hypotheses:

$H_0$. $s = 1$ (renewal process).

$H_a$. $s \neq 1$ (quasi-renewal process).

The computed value of the test statistics $t = -11.8822$ and the rejection region ($t > 4.604$ or $-4.604 < t$) lead to the rejection of $H_0$.

Therefore, we have checked that a quasi-renewal process with ratio $\hat{r} = 1.87706$ is an appropriate model for the given data.

Using the Kolmogorov-Smirnov test, giving a $p$-value of 0.952, and the QQ plot (not shown), which is pretty linear, we have checked that the $Z_i$s are consistent with the normal distribution with mean $\hat{\mu} = 1,322.6$ and standard deviation $\hat{\sigma} = 282.087$.

*4.2.2 Computing the quasi-renewal function and the variance of* N(T) *(see Pham and Wang, 2001).* The quasi-renewal function is equal to:

$$H(T) = \sum_{i=1}^{\infty} P(S_i \leq T), \quad \text{where } S_i = \sum_{k=1}^{i} X_i.$$

Because the first inter-repair time is normally distributed with mean $\hat{\mu} = 1,322.6$ and standard deviation $\hat{\sigma} = 282.087$, the random variables $S_i$ are also normally distributed:

$$Si \sim N\left(\hat{\mu}\,\frac{1-r^i}{1-r},\ \hat{\sigma}^2\,\frac{1-r^{2i}}{1-r^2}\right). \tag{9}$$

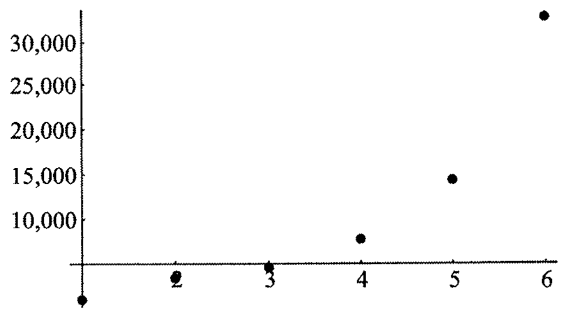


**Figure 3.**
Plot of $\ln(X_i)$ against $i$

Therefore,

$$H(T) = \sum_{i=1}^{\infty} \Phi\left(\left[T - \hat{\mu}\,\frac{1-r^i}{1-r}\right] / {}^{\hat{\sigma}^2 \frac{1-r^{2i}}{1-r^2}}\sqrt{2}\right). \tag{10}$$

In order to compute the variance of $N(T)$ we need $E[N^2(T)]$. In Pham and Wang (2001) it is shown that:

$$E[N^2(T)] = \sum_{i=1}^{\infty} (2i-1)P(S_i \leq T).$$

Therefore,

$$\text{var}[N(T)] = \sum_{i=1}^{\infty} (2i-1)P(S_i \leq T) - H^2(T). \tag{11}$$

*4.2.3 Evaluating the expected warranty cost over the warranty period (0, 36,000).* For the data in Table I, we find $H(T) = 4.71683$ and $\text{var}[N(T)] = 0.2032$. Only seven terms of equations (10) and (11) were needed to achieve an error less than $10^{-5}$.

Using equation (3), and assuming particular values for the cost parameters $a = 1$ and $E[Y] = b = 2.5$, we obtain $E[C(36,000)] = 26.8854$.

In this example we have modelled the inter-repair times by a quasi-renewal process. As expected, the appropriate model was found to be a non-decreasing quasi-renewal process, i.e. the warranty repairs are improved repairs. This is in contrast to maintenance problems, where aging and wear of the item lead to a decreasing trend in the inter-repair times. In warranty, the repaired item is an almost new item and the repair fixes faults that will improve its performance, which will lead to an increasing trend in the inter-repair times.

## 5. Conclusions

In this paper we have presented a framework for the analysis of the lifetime of a unit that undergoes multiple repairs. In our model, each repair affects the lifetime of the repaired unit in an identical way. This approach to modeling imperfect repairs is based on the concepts of the accelerated distribution functions, which leads to the well-known quasi-renewal process or, equivalently, to the geometric process. We have demonstrated a procedure of evaluating the expected warranty cost over the warranty period under free replacement, non-renewing warranty policy. Our future research will address the application of these methods to warranty models with non-zero repair time.

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**Management relevance**

Growth in the use of product warranty as a strategic tool has increased quite significantly over the past decade. A few of the many reasons closely tied to use of warranties by vendors include:

- consumers' rising awareness of quality issues;
- many consumers having neither the time nor the inclination to deal with product failures and repairs; and
- due to the increasing complexity of products, consumers are often unable to judge quality before buying a product, and the warranty is treated as a signal for the quality and reliability of the product.

Whenever competition is tough, profit margins are slim and customer expectations are high, it is increasingly important to analyse and understand the processes related to warranty. With warranty costs running into the billions and customer dissatisfaction on the rise, manufacturers must have effective warranty management. On one hand the customer expects a warranty program, but on the other hand, in order to be successful in the marketplace, the manufacturer needs it.

The warranty contract should be much more than simply fixing product field failure. Using the warranty database the manufacturer can identify issues and opportunities, build customer loyalty and enhance brand reputation. Focus on post-sales services becomes even more important in view of the ratio between the number of units in service and the number of units sold annually.

For example, the number of automobiles in service on the US has grown from 60 million in 1950 to approximately 300 million today, while recent sales are around 17 million per year. Thus, the ratio between of units in service to newly sold units is roughly 17:1. This pattern is common for many industry sectors. If we take a look at the companies that are leading the way in customer satisfaction, making and selling quality products, we see that they are the ones making aggressive changes to post-sales services, starting with their warranty programs.

A product warranty is an agreement offered by a producer to a consumer to replace or repair a faulty item, or to partially or fully reimburse the consumer in the event of a failure.

Warranties are widely used and serve many purposes, including protection for the producer, the seller and the consumer. They are also used as signals of quality and as an element of marketing strategy.

From the buyer's point of view, the main role of warranties in these transactions is protectional. Specifically, the warranty assures the buyer that a faulty item will be either repaired or replaced at no cost or at a reduced cost. A second role of warranties is informational, since they may specify appropriate use, and may be viewed as an indication of quality.

One of the main roles of warranties is from the producer's point of view is also protectional. Warranty terms may and often do specify the use and conditions for use for which the product is intended and provide for limited coverage or no coverage at all in the event of misuse of the product.

A second important purpose of warranties for the seller is promotional. As buyers often infer a more reliable product when a long warranty is offered, this has been used as an effective advertising tool. In addition, warranty has become an instrument used in competition with other manufacturers in the marketplace, similarly to product performance and price.

Despite the fact that warranties are so commonly used, the accurate pricing of warranties in many situations remain an unsolved problem. This may seem surprising, since the fulfillment of warranty claims may cost companies large amounts of money. Underestimating true warranty costs will result in losses for a company. On the other hand, overestimating true warranty costs will result in uncompetitive product prices.

The common warranty parameters of interest to be analyzed and evaluated are the expected total warranty cost and the warranty cost per unit time over the warranty period for a particular item, as well as the lifecycle of the product. These quantities summarize the financial risk or burden carried by buyers, sellers and decision makers.

In this study we analyse the failure process and related imperfect warranty repairs under non-renewing free replacement warranty policy. We model the imperfect warranty repairs using a quasi-renewal process and evaluate the corresponding expected warranty cost. We illustrate our findings using data from an existing warranty database.

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